

### Biot - Savart LAW :

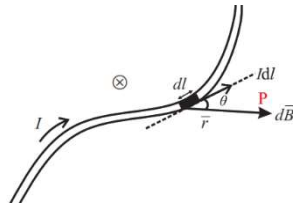
Consider arbitrarily shaped wire carrying a current  $I$ . Let  $dl$  be a length segment along the wire. The field  $dB$  at a distance  $r$  from the length segment is

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

where  $\theta$  is the angle between  $\vec{dl}$  and  $\vec{r}$ . In vector form,

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \vec{r}}{r^3}$$

Direction of  $\vec{dB}$  is given by the cross product of  $\vec{dl} \times \vec{r}$



### Magnetic field due to a current carrying straight infinite (long) wire

We need to find the magnetic field  $\vec{B}$  at a point P which is at a perpendicular distance of  $R$  from the wire. Let us consider a small current element  $I dl$  at a distance  $r$  from P

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

and the direction as given by  $\vec{dl} \times \vec{r}$  is into the paper. Assuming symmetry of the upper half and lower half

$$B = 2 \int_0^\infty dB = 2 \frac{\mu_0}{4\pi} \int_0^\infty \frac{I dl \sin\theta}{r^2}$$

$$\text{BUT } r = \sqrt{l^2 + R^2} \text{ and } \sin(180 - \theta) = \sin\theta = \frac{R}{r} = \frac{R}{\sqrt{l^2 + R^2}}$$

$$\text{Thus, } B = \frac{\mu_0 I}{2\pi} \int_0^\infty \frac{dl}{l^2 + R^2} \cdot \frac{R}{\sqrt{l^2 + R^2}} = \frac{\mu_0 IR}{2\pi} \int_0^\infty \frac{dl}{(l^2 + R^2)^{3/2}}$$

$$B = \frac{\mu_0 IR}{2\pi} \frac{1}{R^2} = \frac{\mu_0 I}{2\pi R}$$

NOTE: If you want to consider for only one half then  $B = \frac{\mu_0 I}{4\pi R}$

Thus the field is inversely proportional to the distance from the wire

### Magnetic field due to a current carrying Circular arc of a wire

Consider a circular arc AB, carrying a current  $I$  and subtends an angle  $\theta$  at the center of the circle O, of which this arc is a part. Let  $r$  be the radius. By Biot-Savart law

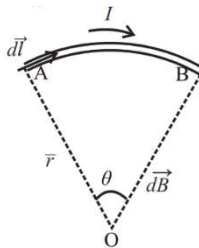
$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2} = \frac{\mu_0}{4\pi} \frac{I dl \sin 90}{r^2} = \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

The direction is into the plane of the paper at O. Total field at O is

$$B = \int dB = \frac{\mu_0}{4\pi} \int_A^B \frac{I dl}{r^2} = \frac{\mu_0}{4\pi} \int_0^\theta \frac{I r d\theta}{r^2} = \frac{\mu_0 I \theta}{4\pi r}$$

Where  $\theta$  is in radians

$$\text{NOTE: For a full circle } B = \frac{\mu_0}{4\pi} \frac{I 2\pi}{r} = \frac{\mu_0 I}{2r}$$



### Axial Magnetic field due to a current carrying Circular loop

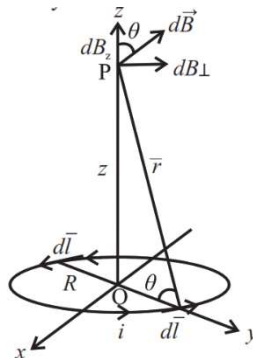
Consider a circular loop in the xy plane, of radius  $R$  and center O, carrying a current  $I$ . We need to calculate the magnetic field at point P which is  $z$  away from the center and along the z-axis. Let us consider an element  $dl$  on the loop and P is  $\vec{r}$  away from  $dl$ .  $r^2 = R^2 + z^2$

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

But  $\theta=90^\circ$ , angle between  $\vec{dl}$  and  $\vec{r}$

$$\text{Thus, } dB = \frac{\mu_0}{4\pi} \frac{I dl}{r^2} = \frac{\mu_0}{4\pi} \frac{I dl}{R^2 + z^2}$$

$\vec{dB}$  is perpendicular to  $\vec{dl}$  and  $\vec{r}$ . Resolving  $dB$  into its components  $dB_z = dB \cos\theta$  and  $dB_\perp$  along z-axis and perpendicular to z-axis respectively. For all such elements on the ring the sum of all  $dB_\perp$  will cancel out. Hence only z components remain



$$\text{Thus, } B = B_z = \int dB_z = \frac{\mu_0}{4\pi} I \int \frac{dl \cos\theta}{Z^2 + R^2}$$

$$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{Z^2 + R^2}}$$

$$\text{Thus, } B_z = \frac{\mu_0}{4\pi} I \int \frac{R dl}{(Z^2 + R^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{IR}{(Z^2 + R^2)^{3/2}} \cdot 2\pi R$$

$$B_z = \frac{\mu_0}{2} \frac{IR^2}{(Z^2 + R^2)^{3/2}}$$

NOTE:

If  $z=0$  (at the center point O),  $B = \frac{\mu_0 I}{2R}$  and  $\frac{\mu_0 NI}{2R}$  if there are  $N$  turns

$$\text{For a very large distance } z \gg R, B_z = \frac{\mu_0 IR^2}{2z^3} = \frac{\mu_0 IA}{2\pi z^3} = \frac{\mu_0 m}{2\pi z^3}$$

where  $A=\pi R^2$  and  $m=\text{magnetic moment} = IA$

In vector form  $\vec{m} = I\vec{A}$  and direction of  $\vec{A}$  is perpendicular to  $A$

$$\vec{B}_z = \frac{\mu_0 \vec{m}}{2\pi z^3}, \text{ Both } \vec{B}_z \text{ and } \vec{m} \text{ are perpendicular to the plane of the loop}$$

### Force on Two long current carrying parallel wires:

Consider two long parallel straight conductors with currents  $I_1$  and  $I_2$  flowing through them as shown. The magnetic field due to wire 1 on 2 will be

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force by Lorentz law will be

$$F_{21} = B_1 I_2 \int dl$$

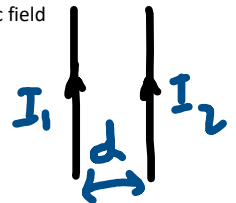
For  $L$  length of wire  $F_{21} = B_1 I_2 L$

$$\text{Force per unit length of wire 2} = BI_2 = \frac{\mu_0 I_1 I_2}{2\pi d}$$

$$\text{Similarly Force per unit length on wire 1 will be } = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Both these forces will be attractive in nature.

If the currents were in opposite direction then the magnitude of the forces will be the same but they would be repulsive in nature.



### Electric and Magnetic Force

Electric Force  $F_e = qE$  where  $E$  is the electric field

Magnetic force  $F_m = qvB \sin\theta$  where  $v$  is the velocity of the charge  $q$  in a magnetic field  $B$  and  $\theta$  is the angle between  $\vec{v}$  and  $\vec{B}$ .

In vector form,  $\vec{F}_m = q \vec{v} \times \vec{B}$

Thus for magnetic force to exist, the charge must be in motion in a magnetic field. The direction of this force is given by Fleming's Left hand rule and is perpendicular to both  $\vec{v}$  and  $\vec{B}$ . Since the magnetic force is perpendicular to displacement, hence it **does no work**. Thus, magnetic force can change the direction of the particle but not its speed.

NOTE: Unit of  $B$  is Tesla (T).

$$1 \text{ T} = 10^4 \text{ gauss}$$

### EXTRA:

$$F_m = qvB \sin\theta = It v B \sin\theta = I \frac{L}{v} v B \sin\theta = BIL \sin\theta$$

$$F_m = BIL \text{ if } \theta = 90^\circ, \text{ angle between } \vec{v} \text{ and } \vec{B}$$

